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Attorney's Docket No. 042933/298965

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant: Nativadade Lobo

Confirmation No.: 5615

Appl. No.: 09/625,201

Filed: July 21, 2000

For: PULSE SHAPING WHICH COMPENSATES FOR COMPONENT
DISTORTION

BOX ISSUE FEE

Commissioner for Patents

P.O. Box 1450

Alexandria, VA 22313-1450

SUBMITTAL OF PRIORITY DOCUMENT

To complete the requirements of 35 U.S.C. § 119, enclosed is a certified copy of Great Britain priority Application No. 9801308.9, filed January 21, 1998.

Respectfully submitted,

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Registration No. 59,932

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Tamara Stevens

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Cardiff Road
Newport
South Wales
NP10 8QQ

I, the undersigned, being an officer duly authorised in accordance with Section 74(1) and (4) of the Deregulation & Contracting Out Act 1994, to sign and issue certificates on behalf of the Comptroller-General, hereby certify that annexed hereto is a true copy of the documents as originally filed in connection with patent application GB9801308.9 filed on 21 January 1998.

In accordance with the Patents (Companies Re-registration) Rules 1982, if a company named in this certificate and any accompanying documents has re-registered under the Companies Act 1980 with the same name as that with which it was registered immediately before re-registration save for the substitution as, or inclusion as, the last part of the name of the words "public limited company" or their equivalents in Welsh, references to the name of the company in this certificate and any accompanying documents shall be treated as references to the name with which it is so re-registered.

In accordance with the rules, the words "public limited company" may be replaced by p.l.c., plc, P.L.C. or PLC.

Re-registration under the Companies Act does not constitute a new legal entity but merely brings the company to certain additional company law rules.

Signed



Dated 21 August 2007

Request for grant of a patent

(See the notes on the back of this form. You can also get an explanatory leaflet from the Patent Office to help you fill in this form)

The Patent Office

Cardiff Road
Newport
Gwent NP9 1RH



1. Your reference

PAT 98001 GB

2. Patent application number

(The Patent Office will fill in this part) 21 JAN 1998

9801308.9

3. Full name, address and postcode of the or of each applicant (underline all surnames)

NOKIA MOBILE PHONES LIMITED
KEILALAHDENTIE 4
02150 ESPOO
FINLAND

Patents ADP number (if you know it)

If the applicant is a corporate body, give the country/state of its incorporation

FINLAND

5911995004

4. Title of the invention

RECEIVER/MODULATOR

5. Name of your agent (if you have one)

MRS HELEN LOUISE HAWS

"Address for service" in the United Kingdom to which all correspondence should be sent (including the postcode)

NOKIA MOBILE PHONES
PATENT DEPARTMENT
ST GEORGES COURT
ST GEORGES ROAD
CAMBERLEY
SURREY GU15 3QZ

UK

Patents ADP number (if you know it)

4105177001

6. If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and (if you know it) the or each application number

Country

Priority application number
(if you know it)

Date of filing
(day / month / year)

7. If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date of the earlier application

Number of earlier application

Date of filing
(day / month / year)

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? (Answer 'Yes' if:

- a) any applicant named in part 3 is not an inventor, or
 - b) there is an inventor who is not named as an applicant, or
 - c) any named applicant is a corporate body.
- See note (d))

YES

I certify this to be a true copy.

G D Court
Acting for Comptroller

Patents Form 1/77

9. Enter the number of sheets for any of the following items you are filing with this form. Do not count copies of the same document

Continuation sheets of this form

Description

24

Claim(s)

Abstract

Drawing(s)

IN WITH TEXT

10. If you are also filing any of the following, state how many against each item.

Priority documents

Translations of priority documents

Statement of inventorship and right to grant of a patent (*Patents Form 7/77*)

Request for preliminary examination and search (*Patents Form 9/77*)

Request for substantive examination (*Patents Form 10/77*)

Any other documents
(please specify)

11.

I/We request the grant of a patent on the basis of this application.

Signature

H L HAWS

AGENT FOR THE APPLICANT

Date 21/1/98

12. Name and daytime telephone number of person to contact in the United Kingdom

HELEN HAWS 01276 419346

Warning

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Notes

- If you need help to fill in this form or you have any questions, please contact the Patent Office on 0645 500505.
- Write your answers in capital letters using black ink or you may type them.
- If there is not enough space for all the relevant details on any part of this form, please continue on a separate sheet of paper and write "see continuation sheet" in the relevant part(s). Any continuation sheet should be attached to this form.
- If you have answered 'Yes' Patents Form 7/77 will need to be filed.
- Once you have filled in the form you must remember to sign and date it.
- For details of the fee and ways to pay please contact the Patent Office.

```
Needs["LaurentFunctions`"]
```

```
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule  
PhaseAngle[L_][t_] => (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).
```

```
Needs["LaurentNotationTest`"]
```

```
Needs::nocont : Context LaurentNotationTest' was not created when Needs was evaluated.
```

We develop the modulating function that is used in GSM first to use as an example to demonstrate Laurent's idea.

```
T :=  $\frac{3}{812500}$   
BT := 0.3
```

```
ModulationIndex :=  $\frac{1}{2}$ 
```

See LaurentTheory.nb to see the derivation of the C functions

When running the notebook we fix the value of L here and the rest of the graphs and functions use this value

For wide band CDMA applications
efficient operation of the P.A. occurs when it is saturated
and linear. If the P.A. is not efficient
+ this power of hand held transmitters is a
problem.

- By approximating to a constant
envelope modulation we can
mitigate the effects of the non-linear
P.A. We show that this can be
done ~~without a penalty~~.

```

L := 6;
PhaseFunction[t_] = N[ $\phi_{L,t}$ ];
phasepoints = Table[{t, PhaseFunction[t T]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 =  $-1.91796 \times 10^{-7}$ .

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 =  $-5.52869 \times 10^{-7}$ .

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 =  $-1.96696 \times 10^{-7}$ .

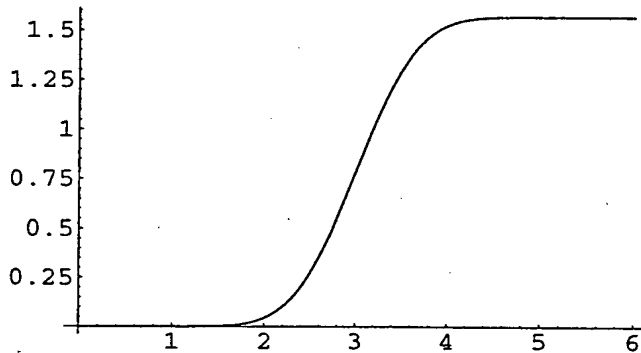
General::stop :
Further output of NIntegrate::ncvb will be suppressed during this calculation.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

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of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

General::stop :
Further output of NIntegrate::slwcon will be suppressed during this calculation.

```



- Graphics -

```

BitSeq = Table[1, {i, 1, 20}]
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

RandomBitSeq = Table[Random[Integer, {0, 1}], {i, 1, 40}] // Map[# (-2) + 1 &, #] &
{1, 1, -1, -1, -1, 1, 1, -1, 1, 1, 1, 1, 1, -1, 1, -1, 1, 1, -1, 1, -1, -1, -1,
-1, 1, 1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, -1, -1, -1, 1}

```

Testing The Modulator ...

Calculating the bandwidth of the signal

```

x11 = Table[LaurentC[L][0][t T], {t, 0, L + 1 - 1/200, 1/200}];
((Drop[x11, -1] - Drop[x11, 1]) 200)^2 / 200 // Apply[Plus, #] &
1.29684

```

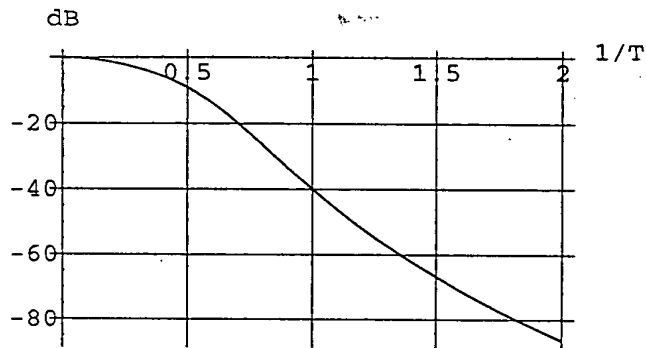
Here the pulse 6T is just stretched to 8T to see the performance

Example of stretching a C_0 pulse Of 6 to 8

In this section we consider filtering the pulse

```
fil = BesselFilter[8][LowPass3dB[1/T 2 Pi 0.3]][FrequencyResponse][s];
```

```
Plot[20 Log[10, Evaluate[(fil /. s -> 2 Pi f / T) // Abs]], {f, 0, 2},
GridLines -> Automatic, AxesLabel -> {"1/T", "dB"}]
```

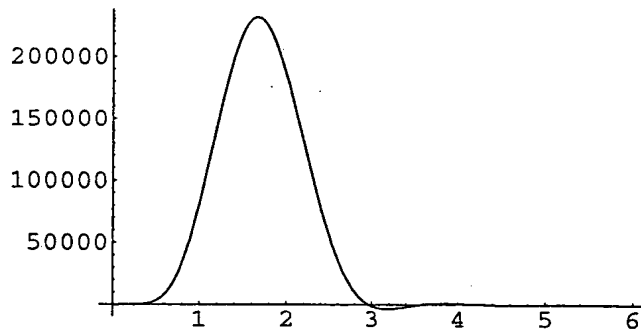


- Graphics -

```
Res[t_] = InverseLaplaceTransform[fil, s, t]
```

```
2027025 ((9.36812 + 0. I) E-897172. t Cos[139301. t] -
(13.7944 + 0. I) E-835672. t Cos[420044. t] + (4.73111 + 0. I) E-701358. t Cos[708768. t] -
(0.304883 + 0. I) E-455818. t Cos[1.02016 × 106 t] +
(29.1264 + 0. I) E-897172. t Sin[139301. t] -
(9.89398 + 0. I) E-835672. t Sin[420044. t] - (0.321205 + 0. I) E-701358. t Sin[708768. t] +
(0.375154 + 0. I) E-455818. t Sin[1.02016 × 106 t])
```

```
Plot[Res[t T], {t, 0, 6}, PlotRange -> All]
```



- Graphics -

```
filTaps = Table[Res[t T], {t, 0, 6, 1/20}] // Chop;
```

We estimate the group delay to be $1.8T$

```
Length[filTaps]
```

```
121
```

```

L := 8;
PhaseFunction[t_] = N[ $\phi_L$ , t];
phasepoints = Table[{t, PhaseFunction[t]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions'Private't1 near LaurentFunctions'Private't1 =  $-3.68196 \times 10^{-7}$ .

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions'Private't1 near LaurentFunctions'Private't1 =  $-7.4914 \times 10^{-7}$ .

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
in LaurentFunctions'Private't1 near LaurentFunctions'Private't1 =  $-5.43232 \times 10^{-7}$ .

General::stop :
Further output of NIntegrate::ncvb will be suppressed during this calculation.

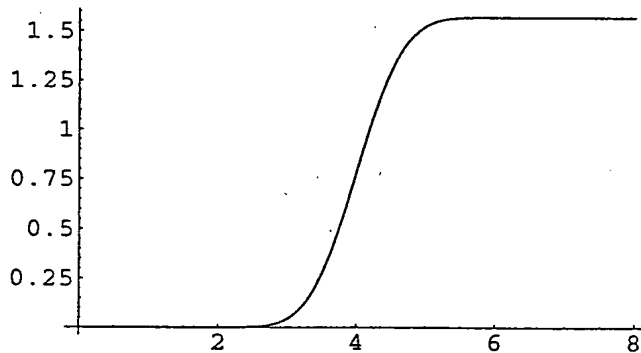
NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
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If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

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of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

General::stop :
Further output of NIntegrate::slwcon will be suppressed during this calculation.

```



- Graphics -

```

pulse = Table[LaurentC[8][0][t T], {t, 0, 9, 1/20}];
pulse2 = Table[LaurentC[8][1][t T], {t, 0, 7, 1/20}];

```



```
FilteredPulse = T/20 MapConvolve[pulse, filTaps] // Take[#, {34, Length[#]}]&
```

```
{1.73731×10-15, 5.35887×10-15, 1.60413×10-14, 4.66278×10-14, 1.31688×10-13,
3.61578×10-13, 9.65729×10-13, 2.51044×10-12, 6.35507×10-12, 1.56747×10-11,
3.76889×10-11, 8.83867×10-11, 2.02275×10-10, 4.51958×10-10, 9.86455×10-10,
2.10426×10-9, 4.38922×10-9, 8.95697×10-9, 1.78914×10-8, 3.4999×10-8, 6.70842×10-8,
1.26054×10-7, 2.32319×10-7, 4.20166×10-7, 7.46062×10-7, 1.30124×10-6,
2.23035×10-6, 3.75853×10-6, 6.23×10-6, 0.0000101618, 0.0000163174, 0.0000258049,
0.0000402068, 0.000061746, 0.000093496, 0.00013964, 0.000205786, 0.000299337,
0.000429924, 0.000609895, 0.000854853, 0.00118424, 0.00162196, 0.00219697,
0.0029439, 0.00390362, 0.00512368, 0.00665873, 0.00857068, 0.0109288, 0.0138093,
0.0172954, 0.021476, 0.0264451, 0.0323001, 0.0391407, 0.0470666, 0.0561754,
0.0665603, 0.0783079, 0.0914948, 0.106186, 0.12243, 0.140261, 0.159691, 0.180711,
0.20329, 0.22737, 0.25287, 0.279682, 0.307673, 0.336686, 0.366539, 0.397029,
0.427934, 0.459013, 0.490013, 0.520668, 0.550708, 0.579856, 0.607837, 0.634379,
0.65922, 0.682107, 0.702805, 0.721098, 0.73679, 0.749713, 0.759727, 0.76672,
0.770616, 0.771369, 0.768969, 0.763442, 0.754846, 0.743274, 0.728852, 0.711736,
0.692108, 0.670178, 0.646176, 0.62035, 0.592964, 0.56429, 0.534607, 0.504193,
0.473326, 0.442277, 0.411305, 0.380656, 0.350558, 0.321221, 0.29283, 0.26555,
0.239519, 0.214852, 0.191636, 0.169936, 0.149791, 0.13122, 0.114217, 0.0987613,
0.0848118, 0.0723143, 0.0612016, 0.0513967, 0.0428146, 0.035365, 0.0289542,
0.023487, 0.0188687, 0.0150065, 0.0118109, 0.00919671, 0.00708412, 0.0053992,
0.00407439, 0.00304877, 0.00226811, 0.0016848, 0.00125761, 0.000951417, 0.000736739,
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0.000281223, 0.000251974, 0.000220764, 0.000187441, 0.000152497, 0.000116824,
0.0000815146, 0.0000477044, 0.0000164566, -0.0000113198, -0.0000349206,
-0.0000538729, -0.0000679391, -0.0000771062, -0.0000815622, -0.0000816651,
-0.0000779073, -0.000070877, -0.000061222, -0.0000496142, -0.0000367186,
-0.0000231668, -9.53584×10-6, 3.66817×10-6, 0.0000160205, 0.0000271838,
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2.3199×10-6, 1.71859×10-6, 1.25466×10-6, 9.0236×10-7, 6.39109×10-7,
4.45602×10-7, 3.05722×10-7, 2.06318×10-7, 1.36898×10-7, 8.92721×10-8,
5.71866×10-8, 3.59686×10-8, 2.22017×10-8, 1.34418×10-8, 7.97812×10-9,
4.63953×10-9, 2.64191×10-9, 1.4722×10-9, 8.02333×10-10, 4.27388×10-10,
2.2234×10-10, 1.12886×10-10, 5.59276×10-11, 2.70717×10-11, 1.28211×10-11,
5.92683×10-12, 2.65259×10-12, 1.14156×10-12, 4.73562×10-13, 1.94329×10-13,
7.95416×10-14, 3.14517×10-14, 1.23089×10-14, 4.73302×10-15, 1.66595×10-15,
4.65283×10-16, 1.93314×10-16, 9.87738×10-17, 2.31472×10-18, -1.50823×10-17,
-1.01228×10-17, -6.49877×10-18, -2.08455×10-18, 1.80177×10-19, 1.70062×10-19,
1.9917×10-19, -1.14008×10-19, -1.66045×10-20, 3.7877×10-21, 1.47752×10-21,
1.94753×10-21, 3.72298×10-22, 2.02535×10-22, 1.36007×10-22, 6.72514×10-24,
7.07296×10-23, 6.84862×10-23, 3.68916×10-23, 2.6013×10-24, -1.51251×10-24,
-5.51867×10-25, 7.52915×10-26, 5.81584×10-25, 1.75371×10-25, 1.22392×10-26, 0}
```

```

FilteredPulse2 = T/20 MapConvolve[pulse2, filTaps] // Take[#, {50, Length[#]}]&

{1.27972×10-7, 2.18306×10-7, 3.66972×10-7, 6.0806×10-7, 9.93427×10-7, 1.60076×10-6,
2.54471×10-6, 3.99199×10-6, 6.18149×10-6, 9.45059×10-6, 0.0000142688, 0.0000212804,
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7.24522×10-6, 7.16306×10-6, 6.89804×10-6, 6.49223×10-6, 5.98586×10-6,
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4.82884×10-7, 3.59854×10-7, 2.64486×10-7, 1.91715×10-7, 1.37044×10-7,
9.65984×10-8, 6.71324×10-8, 4.59913×10-8, 3.10539×10-8, 2.06612×10-8, 1.3542×10-8,
8.74155×10-9, 5.55593×10-9, 3.47564×10-9, 2.13956×10-9, 1.29613×10-9,
7.73129×10-10, 4.54101×10-10, 2.62031×10-10, 1.47888×10-10, 8.15113×10-11,
4.40884×10-11, 2.37341×10-11, 1.26865×10-11, 6.58958×10-12, 3.34992×10-12,
1.6239×10-12, 7.01037×10-13, 2.33036×10-13, 7.75051×10-14, -3.22249×10-15,
-8.12875×10-14, -7.89371×10-14, -5.29088×10-14, -3.14297×10-14, -8.21475×10-15,
4.12557×10-15, 1.9906×10-15, 1.82068×10-15, -4.36969×10-15, 4.67143×10-16,
7.65162×10-16, 5.67707×10-16, 4.93342×10-16, 7.52562×10-17, 7.75606×10-17,
4.50401×10-17, -2.18469×10-17, 1.39012×10-16, 1.92164×10-16, 6.31946×10-17,
-6.72328×10-18, 1.24704×10-17, 1.42063×10-17, 7.33335×10-18, 1.13717×10-19,
3.90929×10-19, 1.30285×10-19, 0}

FiltPulse[8][0] = {(Table[t T, {t, 0, 20, 1/20}] // N) //
Take[#, Length[FilteredPulse]]&, FilteredPulse} // Transpose //
Interpolation

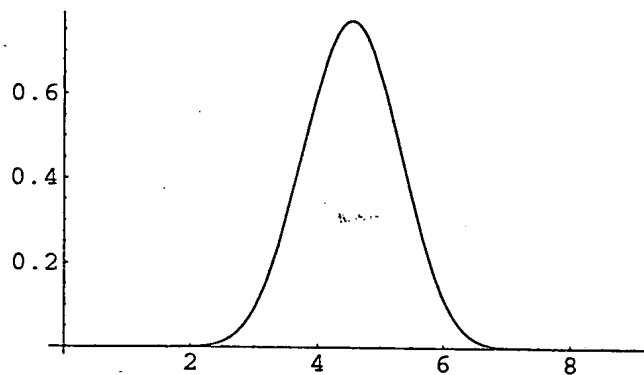
InterpolatingFunction[{{0, 0.0000492923}}, <>]

FiltPulse[8][1] = {(Table[t T, {t, 0, 20, 1/20}] // N) //
Take[#, Length[FilteredPulse2]]&, FilteredPulse2} // Transpose //
Interpolation

InterpolatingFunction[{{0, 0.0000389538}}, <>]

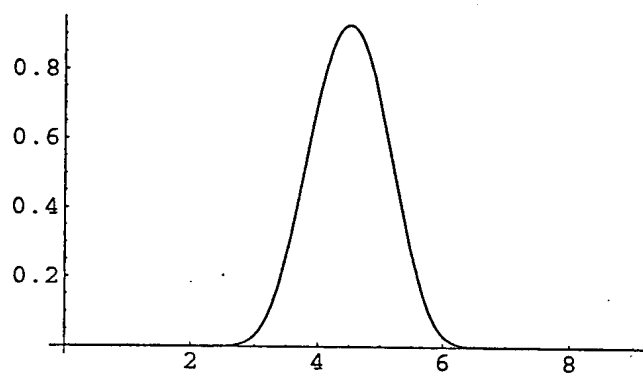
```

```
Plot[FiltPulse[8][0][t T], {t, 0, 9}, PlotRange -> All]
```



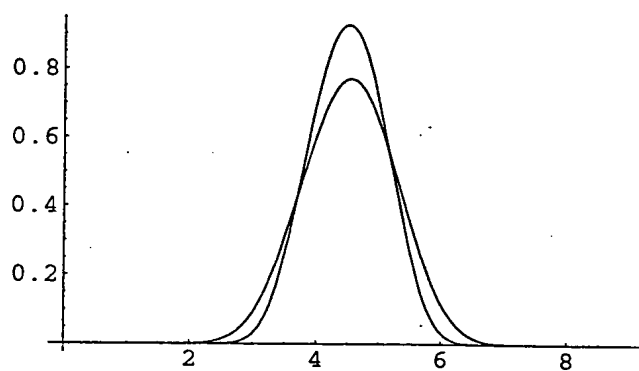
- Graphics -

```
Plot[LaurentC[8][0][t T], {t, 0, 9}, PlotRange -> All]
```



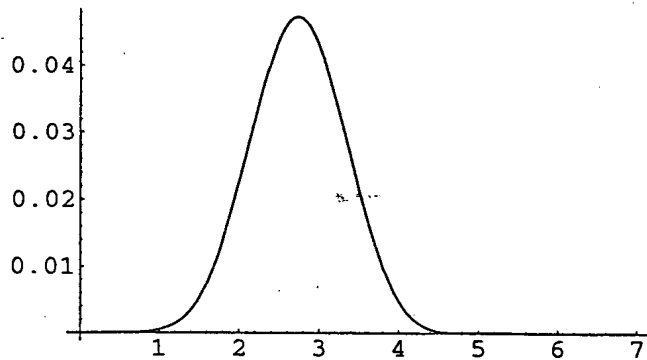
- Graphics -

```
Show[%, %]
```



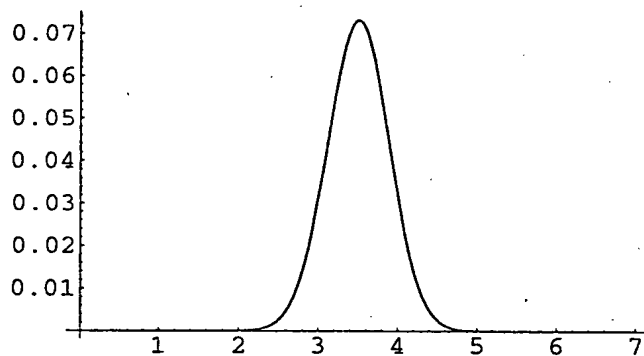
- Graphics -

```
Plot[FiltPulse[8][1][t T], {t, 0, 7}, PlotRange -> All]
```



- Graphics -

```
Plot[LaurentC[8][1][t T], {t, 0, 7}, PlotRange -> All].
```



- Graphics -

```
BandWidth[TestPulse_, Supp_] :=  
Module[{x11}, x11 = Table[TestPulse[t T], {t, 0, Supp - 1/200, 1/200}];  
((Drop[x11, -1] - Drop[x11, 1]) 200)^2 / 200 // Apply[Plus, #]&]
```

```
BandWidth[FiltPulse[8][0], 9]
```

```
0.711659
```

```
BandWidth[LaurentC[8][0], 9]
```

```
1.29684
```

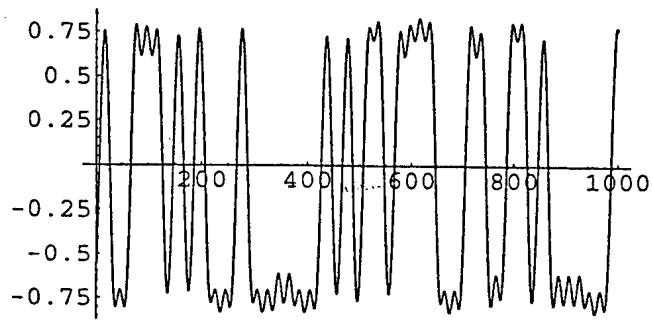
```
BandWidth[FiltPulse[8][1], 7]
```

```
0.00338446
```

```
tom = Modulator[L][Table[1, {i, 1, 100}], SamplingInterval -> T/10,  
NumberOfCurves -> 2, ModulatingPulse -> FiltPulse];
```

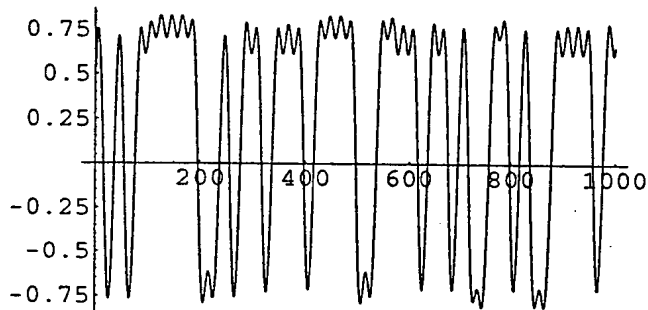
```
tom = Modulator[L][RandomBitSeq, SamplingInterval -> T/10, NumberOfCurves -> 2,  
ModulatingPulse -> FiltPulse];
```

```
ListPlot[Im[tom], PlotJoined -> True]
```



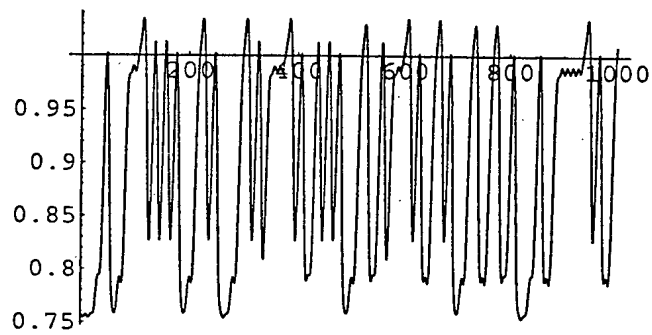
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



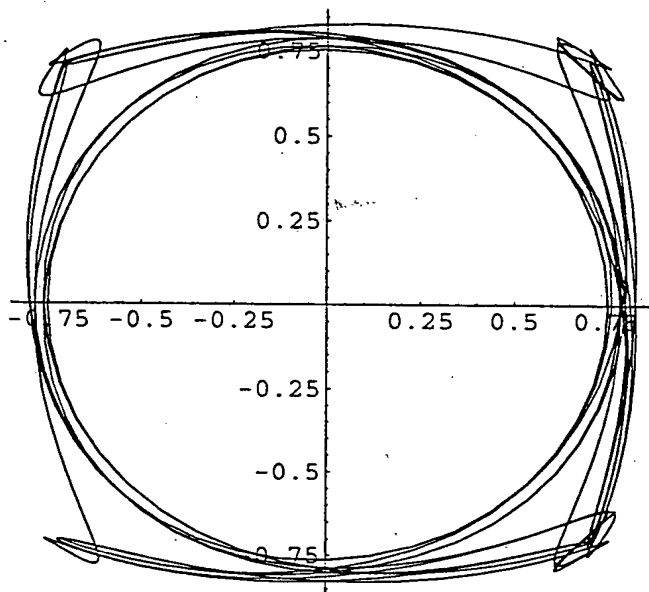
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

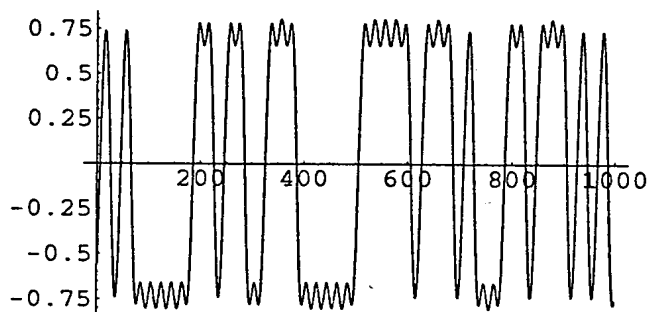
```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

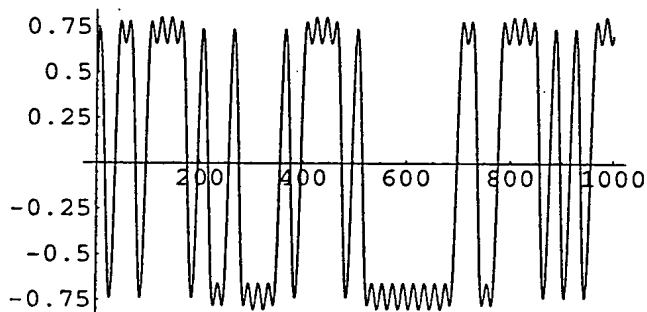
```
tom = Modulator[L][RandomBitSeq, SamplingInterval -> T/10, NumberOfCurves -> 1,  
  ModulatingPulse -> FiltPulse];
```

```
ListPlot[Im[tom], PlotJoined -> True]
```



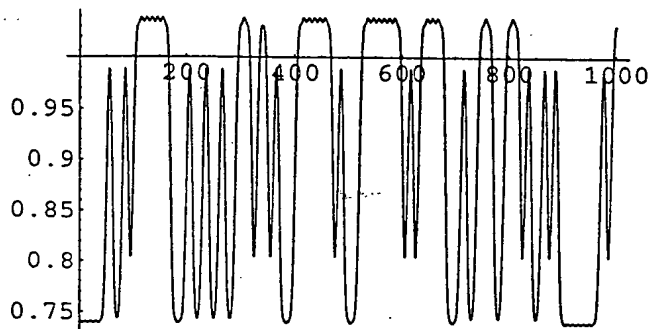
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



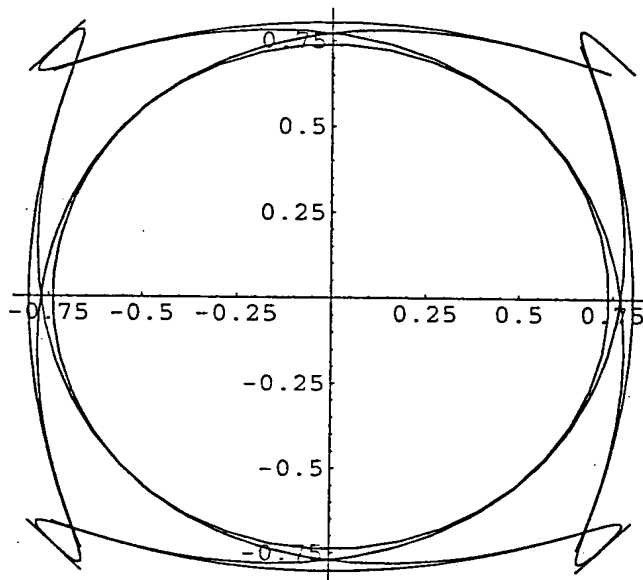
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

```
x11 = Table[TestPulse[6][0][t T], {t, 0, 7 - 1/200, 1/200}];
```

```
((Drop[x11, -1] - Drop[x11, 1]) 200)^2 / 200 // Apply[Plus, #]&
```

```
0.92632
```

```
TestPulse[6][0][t_] = LaurentC[4][0][t 5/7]
```

$$1. \sin\left[\psi\left[4, \frac{3}{812500} + \frac{5t}{7}\right]\right] \sin\left[\psi\left[4, \frac{3}{406250} + \frac{5t}{7}\right]\right] \sin\left[\psi\left[4, \frac{9}{812500} + \frac{5t}{7}\right]\right] \\ \sin\left[\psi\left[4, \frac{5t}{7}\right]\right]$$

```

L := 5;
PhaseFunction[t_] =  $\phi_{L,t}$ ;
phasepoints = Table[{t, PhaseFunction[t]}] // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7
recursive bisections in t1 near t1 =  $-1.48053 \times 10^{-6}$ .

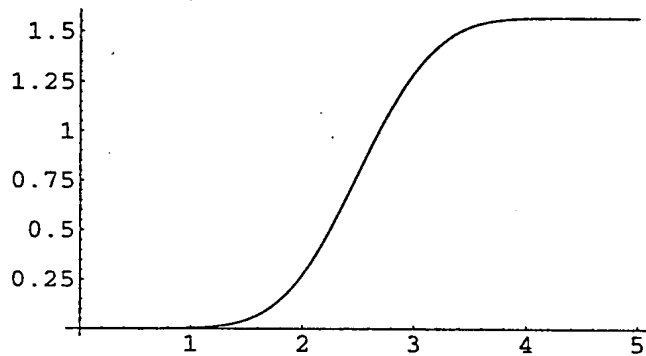
NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7
recursive bisections in t1 near t1 =  $-1.12255 \times 10^{-6}$ .

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7
recursive bisections in t1 near t1 =  $-1.08808 \times 10^{-6}$ .

General::stop :
Further output of NIntegrate::ncvb will be suppressed during this calculation.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

```



- Graphics -

```
L := 6
```

```
TestPulse2[6][0][t_] = LaurentC[4][0][t 6/7]
```

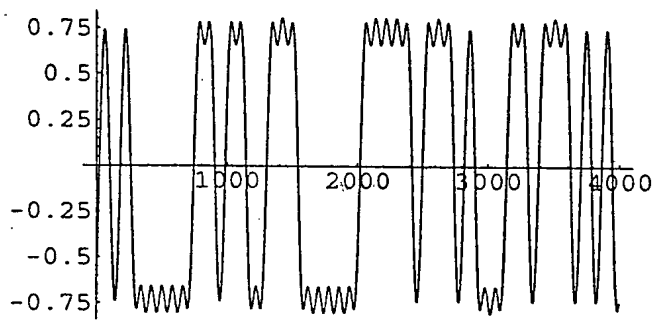
$$1. \sin\left[\psi\left[4, \frac{3}{812500} + \frac{6t}{7}\right]\right] \sin\left[\psi\left[4, \frac{3}{406250} + \frac{6t}{7}\right]\right] \sin\left[\psi\left[4, \frac{9}{812500} + \frac{6t}{7}\right]\right] \\ \sin\left[\psi\left[4, \frac{6t}{7}\right]\right]$$

```
tom = Modulator[L][RandomBitSeq, SamplingInterval -> T/40, NumberOfCurves -> 1,
ModulatingPulse -> FiltPulse];
```

```
Save["ModulatorData.m", {T, L, RandomBitSeq, FiltPulse, tom}]
```

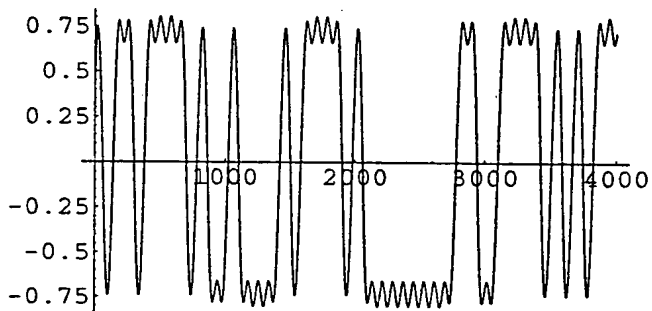


```
ListPlot[Im[tom], PlotJoined -> True]
```



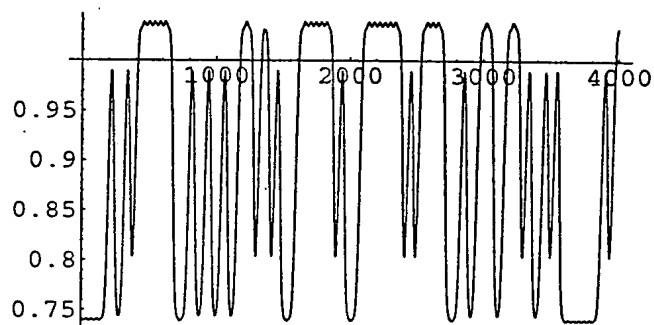
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



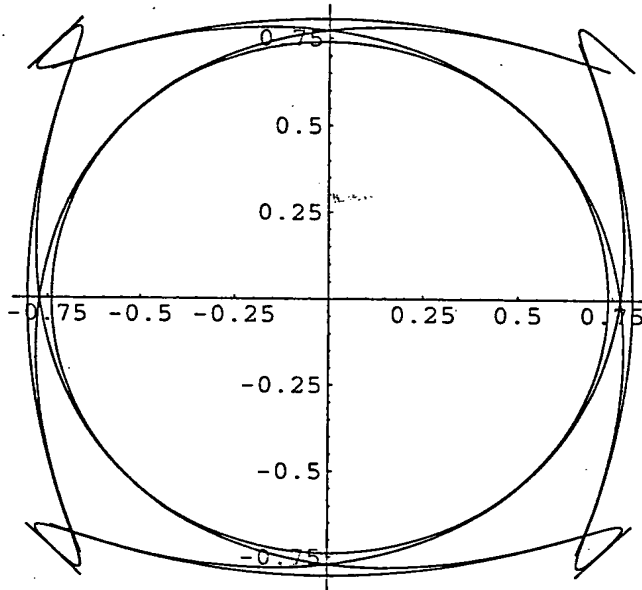
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

```
x11 = Table[TestPulse2[6][0][t T], {t, 0, 7 - 1/200, 1/200}];
```

```
Needs["LaurentFunctions`"]
```

```
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
PhaseAngle[L_][t_] => (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).
```

```
Needs["LaurentNotationTest`"]
```

```
Needs::nocont : Context LaurentNotationTest' was not created when Needs was evaluated.
```

Information on the functions used can be obtained using help.

```
Names["LaurentFunctions`*"]
```

```
{AKN, AlphaKI, ANKInitialStateSetup, BT, FiltPulse, h, hFiltered, InitialState, J,
LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
NumberOfCurves, PhaseAngle, Receiver, ReceiverProper, S, SamplingInterval,
StartingQuadrant, SyncSample, T, C,  $\Phi$ ,  $\psi$ }
```

```
T :=  $\frac{3}{812500}$ 
BT := 0.3
```

```
ModulationIndex :=  $\frac{1}{2}$ 
```

```
<< ModulatorData.m;
```

```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
-1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1,
1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, 1, -1,
1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

We show that it is possible
to build a ~~receiver~~ ^{mobile telephone} to receive ~~the~~
~~pulses that have~~ a signal modulate
using ~~phase~~ super position method
of Laurent - but with different pulses
that have other desirable properties
like low bandwidth etc.

T

$$\frac{3}{812500}$$

$$S_{NT+\Delta T} = \sum_{K=0}^{M-1} \sum_{n'=0}^{L_K-1} J^{A_{KN-n'}} C_{K,n'T+\Delta T}$$

With $L = 8$, and $M = 2$ we can utilise only the main values. It so happens that when $K = 1$, the dominant value occurs at $2.5T$ in the filtered pulse. The three dominant values when $K = 0$ occurs at $3.5T$, $4.5T$ and $5.5T$. The value of the pulse at $6.5T$ is smaller than the value of the pulse with $K = 1$ at $2.5T$. Thus the expression becomes

$$S_{NT+\Delta T} = \sum_{K=0}^1 \sum_{n'=3}^5 J^{A_{KN-n'}} C_{K,n'T+\Delta T}$$

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-3}} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{0,N-4}} \text{Pulse}[0][4T + \delta T] + J^{\alpha_{0,N-5}} \text{Pulse}[0][5T + \delta T] + J^{\alpha_{1,N-2}} \text{Pulse}[1][2T + \delta T]$$

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-3}} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{0,N-4}} \text{Pulse}[0][4T + \delta T] + J^{\alpha_{0,N-5}} \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{0,N-2} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T]$$

Given that α_{N-3} has been already decided, we can precalculate the possibilities and store them.

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-5}} (J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T + \delta T] + \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{N-4} + \alpha_{N-3} + \alpha_{N-2} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T])$$

But $J^2 = 1$ and so we get

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-5}} (J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T + \delta T] + \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T])$$

```
LookUpTable[Pulse_, δT_] :=
Module[{x1, x2, x3, x4},
x1 = {{-1, -1, -1}, {-1, -1, 1},
{-1, 1, -1}, {-1, 1, 1}, {1, -1, -1}, {1, -1, 1}, {1, 1, -1}, {1, 1, 1}};
x2[bitseq_] :=
(J^bitseq[[3]] + bitseq[[2]] Pulse[0][3T + δT] + J^bitseq[[3]] Pulse[0][4T + δT] +
Pulse[0][5T + δT] + J^bitseq[[3]] + bitseq[[1]] Pulse[1][2T + δT]);
{x1, Map[x2[#] &, x1]} // Transpose
```

```
tab = LookUpTable[FiltPulse[8], 0.5 T]
```

```
{{{-1, -1, -1}, -0.00122183 - 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
{{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, 1, 1}, 0.0869922 + 0.770616 I},
{{1, -1, -1}, 0.0869922 - 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
{{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, 1, 1}, -0.00122183 + 0.770616 I}}
```

```
Sort[tab, #1[[1, 3]] > #2[[1, 3]] &]
```

```
{{{1, 1, 1}, -0.00122183 + 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
{{-1, 1, 1}, 0.0869922 + 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
{{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, -1, -1}, 0.0869922 - 0.770616 I},
{{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, -1, -1}, -0.00122183 - 0.770616 I}}
```

```
FiltPulse[8][0][4T + 0.5 T]
```

```
0.770616
```

Now we build a receiver!! First generate the modulated sequence

Options[Modulator]

{StartingQuadrant → 0,
InitialState → {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1},
SamplingInterval → $\frac{3}{26000000}$, NumberOfCurves → 4, ModulatingPulse → LaurentC}

$$\frac{3}{26000000} / T$$

$$\frac{1}{32}$$

tom2 = Modulator[L][RandomBitSeq, NumberOfCurves → 2, ModulatingPulse → FiltPulse];

ListPlot[{Re[tom2], Im[tom2]} // Transpose, PlotJoined → True]

- Graphics -

RandomBitSeq

{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
-1, 1, -1, -1, 1, -1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, -1, -1, -1, 1, -1,
1, 1, 1, -1, 1, -1, -1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}

Receiver[L][tom2, StartingQuadrant → 4]

{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
-1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, 1,
1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, -1,
-1, -1, 1, -1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, -1, -1}

We now calculate the BER given the $\frac{E_b}{N_0}$ and $L = 8$. We write all the possible terms

$J^{a_0, N-5} (J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N)} \text{Pulse}[0][\delta T] +$
 $J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1})} \text{Pulse}[0][T + \delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2})} \text{Pulse}[0][2T + \delta T] +$
 $J^{(a_{N-4}+a_{N-3})} \text{Pulse}[0][3T + \delta T] + J^{a_{N-4}} \text{Pulse}[0][4T + \delta T] + \text{Pulse}[0][5T + \delta T] +$
 $J^{(-a_{N-5})} \text{Pulse}[0][6T + \delta T] + J^{(-a_{N-5}-a_{N-6})} \text{Pulse}[0][7T + \delta T] +$
 $J^{(-a_{N-5}-a_{N-6}-a_{N-7})} \text{Pulse}[0][8T + \delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N-a_{N-1})} \text{Pulse}[1][\delta T] +$
 $J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}-a_{N-2})} \text{Pulse}[1][T + \delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1})} \text{Pulse}[1][2T + \delta T] +$
 $J^{(a_{N-4}+a_{N-3}-a_{N-4})} \text{Pulse}[1][3T + \delta T] + J^{(a_{N-4}-a_{N-5})} \text{Pulse}[1][4T + \delta T] +$
 $J^{(-a_{N-6})} \text{Pulse}[1][5T + \delta T] + J^{(-a_{N-5}+a_{N-7})} \text{Pulse}[1][6T + \delta T])$

We select the imaginary

$J^{a_0, N-5} (J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N)} \text{Pulse}[0][\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2})} \text{Pulse}[0][2T + \delta T] +$
 $J^{a_{N-4}} \text{Pulse}[0][4T + \delta T] + J^{(-a_{N-5})} \text{Pulse}[0][6T + \delta T] +$
 $J^{(-a_{N-5}-a_{N-6}-a_{N-7})} \text{Pulse}[0][8T + \delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}-a_{N-2})} \text{Pulse}[1][T + \delta T] +$
 $J^{(a_{N-4}+a_{N-3}-a_{N-4})} \text{Pulse}[1][3T + \delta T] + J^{(-a_{N-6})} \text{Pulse}[1][5T + \delta T])$

ModulationValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT] :=
(1. I) $x_0+x_1+x_2+x_3+x_4$ Pulse[0][δT] + (1. I) $x_2+x_3+x_4$ Pulse[0][2T + δT] +
(1. I) x_4 Pulse[0][4T + δT] + (1. I) x_5 Pulse[0][6T + δT] +
(1. I) $x_5-x_6-x_7$ Pulse[0][8T + δT] + (1. I) $x_1+x_3+x_4$ Pulse[1][T + δT] +
(1. I) x_3 Pulse[1][3T + δT] + (1. I) x_6 Pulse[1][5T + δT]

[illegible]

```
x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
  Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
  (-I)
```

```
{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}
```

```
Map[(1/2 - σ/2 Erf[ $\frac{\#}{\sigma}$ ])&, x1] // Apply[Plus, #]& // #/128 &;
```

```
Ber[σ_] := Evaluate[%48]
```

```
Ber[0.01]
```

```
0.495
```

```
Erf[Infinity]
```

```
1
```

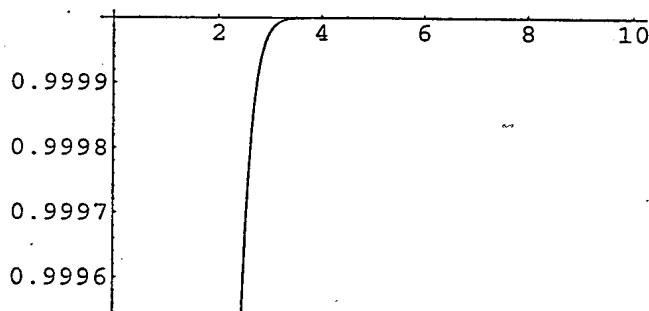
```
D[Erf[x], x]
```

$$\frac{2e^{-x^2}}{\sqrt{\pi}}$$

```
?Erf
```

Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error function erf(z1) - erf(z0).

```
Plot[Erf[x], {x, 0, 10}]
```



- Graphics -

```
?FiltPulse
```

```
Global FiltPulse
```

T := 3 / 812500

Null

?? T

This is the symbol period

?? T

This is the symbol period

T := 3 / 812500

Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]

$J^{(\alpha_{N-4} + \alpha_{N-3} + \alpha_{N-2} + \alpha_{N-1} + \alpha_N)} \text{Pulse}[0][\delta T] + J^{(\alpha_{N-4} + \alpha_{N-3} + \alpha_{N-2})} \text{Pulse}[0][2T + \delta T] +$
 $J^{\alpha_{N-4}} \text{Pulse}[0][4T + \delta T] + J^{(-\alpha_{N-5})} \text{Pulse}[0][6T + \delta T] +$
 $J^{(-\alpha_{N-5} - \alpha_{N-6} - \alpha_{N-7})} \text{Pulse}[0][8T + \delta T] + J^{(\alpha_{N-4} + \alpha_{N-3} + \alpha_{N-1})} \text{Pulse}[1][T + \delta T] +$
 $J^{(\alpha_{N-3})} \text{Pulse}[1][3T + \delta T] + J^{(-\alpha_{N-6})} \text{Pulse}[1][5T + \delta T] /. \{ \alpha_N \rightarrow x0, \alpha_{N-1} \rightarrow x1,$
 $\alpha_{N-2} \rightarrow x2, \alpha_{N-3} \rightarrow x3, \alpha_{N-4} \rightarrow x4, \alpha_{N-5} \rightarrow x5, \alpha_{N-6} \rightarrow x6, \alpha_{N-7} \rightarrow x7 \}$

$(1. I)^{x0 + x1 + x2 + x3 + x4} \text{Pulse}[0][\delta T] + (1. I)^{x2 + x3 + x4} \text{Pulse}[0][2T + \delta T] +$
 $(1. I)^{x4} \text{Pulse}[0][4T + \delta T] + (1. I)^{-x5} \text{Pulse}[0][6T + \delta T] + (1. I)^{-x5 - x6 - x7} \text{Pulse}[0][8T + \delta T] +$
 $(1. I)^{x1 + x3 + x4} \text{Pulse}[1][T + \delta T] + (1. I)^{x3} \text{Pulse}[1][3T + \delta T] + (1. I)^{-x6} \text{Pulse}[1][5T + \delta T]$

$J^{(\alpha_{N-3})} \text{Pulse}[1][3T + \delta T] /. \{ \alpha_N \rightarrow x0, \alpha_{N-1} \rightarrow x1, \alpha_{N-2} \rightarrow x2, \alpha_{N-3} \rightarrow x3,$
 $\alpha_{N-4} \rightarrow x4, \alpha_{N-5} \rightarrow x5, \alpha_{N-6} \rightarrow x6, \alpha_{N-7} \rightarrow x7 \}$

$(1. I)^{x3} \text{Pulse}[1][3T + \delta T]$

$(1. I)^{x3} \text{Pulse}[1][3T + \delta T]$

$(1. I)^{x3} \text{Pulse}[1][3T + \delta T]$

?? J

J = $e^{j\omega}$


```

BeginPackage["LaurentFunctions`"]

T::usage = "This is the symbol period"

BT::usage = "This is the usual product"

h::usage = "This is the raw gaussian pulse"

 $\pi$ ::usage = "This denotes modulation index  $\pi$ "

 $\psi$ ::usage = " $\psi[L,t]$  is Laurents function"

hFiltered::usage = "This is the filtered gaussian pulse"

PhaseAngle::usage = "This function takes some time to calculate. The following
code will draw a graph of the function PhaseFunction[t_] = N[  $\phi_{L,t}$ ];
phasepoints = Table[{t,PhaseFunction[t T]}/N,{t,0,L,1/40}];
ListPlot[phasepoints,PlotJoined -> True]"

S::usage = "S = Sin[ $\pi$ ]"

J::usage = "J =  $e^{j\theta}$ "

C::usage = "C = Cos[ $\pi$ ]"

M::usage = "M =  $2^{L-1}$ "

ModulationIndex::usage = "ModulationIndex = h"

LaurentS::usage = "LaurentS[L][n][t] = Sin[  $\psi[L,t + n T]$ ]/ s"

LaurentC::usage = "LaurentC[L][K][t] is Laurents  $C_{K,t}$ "

AlphaKI::usage = "AlphaKI[LL][K,i] is Laurents  $\alpha_{K,i}$ "

LaurentLK::usage = "LaurentLK[L][K] gives the support of  $C_{K,t}$ "

```

The start of Modulator Definitions

```

ANKInitialStateSetUp::usage =
  "ANKInitialStateSetUp[L][K][InitBitSeq,AccumulatedPhase] sets up the sequence
  of prior states of  $A_{K,N}$  that the modulator went through to get to the
  constellation point specified by AccumulatedPhase which is really  $A_{0,0}$ "

AKN::usage = "AKN[L][K][{State,AccumulatedPhase}] defines  $A_{K,N}$  in terms of  $A_{0,N}$ "

ModulatingPulse::usage =
  "The Pulse is assumed to have the following structure Pulse[L][K][t]"

NumberOfCurves::usage = "The number of pulses used by the modulator"

SamplingInterval::usage =
  "SamplingInterval is the interval between samples of the output of the modulator"

InitialState::usage = "InitialState is the set of bits that are assumed
to be present before i.e {a1,a2,...}"

StartingQuadrant::usage = "StartingQuadrant =  $A_{0,-1}$  and is a number"

Modulator::usage = "Modulator[L][BitSeq,Opts] assumes the following
default options StartingQuadrant -> 0,InitialState -> Table[1,{i,
1,20}],SamplingInterval -> T/32,NumberOfCurves -> 4,ModulatingPulse-
LaurentC . The Pulse is assumed to have the following structure Pulse[L][K][t]"

```

Start of the Receiver Functions

$$\text{LaurentS}[L_][n_][t_] := \text{Sin}[\psi[L, t + nT]] / S$$

```

AlphaKI[LL_][K_, i_] /; (0 < i < LL) && (0 <= K < 2LL-1) :=
Module[{x1, x2, x3, KNum}, x1 := {x2 = Mod[KNum, 2]; KNum =  $\frac{KNum - x2}{2}$ ; x2};
KNum = K; x3 = Table[x1, {ii, 0, LL - 1}]; x3[[i]]

LaurentLK[L_][K_] :=
Module[{x1}, x1 = Table[L (2 - AlphaKI[L][K, ii]) - ii, {ii, 1, L - 1}]; Min[x1]]

LaurentC[L_][K_][t_] /; 0 <= K < 2L :=
LaurentS[L][0][t]  $\prod_{ii=1}^{L-1}$  LaurentS[L][ii + L AlphaKI[L][K, ii]][t]

```

The start of the modulator function

```

ANKInitialStateSetup[L_][K_][InitBitSeq_, AccumulatedPhase_] :=
Module[{x1, x2, x3, x4, x5, acuphase, initbitseq},
initbitseq = InitBitSeq;
acuphase = AccumulatedPhase;
UpdateSeq :=
Module[{}, x1 = acuphase - Sum[initbitseq[[i]] AlphaKI[L][K, i], {i, 1, L - 1}];
acuphase = acuphase - First[initbitseq]; initbitseq = Rest[initbitseq]; x1];
Table[UpdateSeq, {i, 1, LaurentLK[L][K]}]]

AKN[L_][K_] [{State_, AccumulatedPhase_}] :=
AccumulatedPhase - Sum[State[[i + 1]] AlphaKI[L][K, i], {i, 1, L - 1}]

Options[Modulator] := {StartingQuadrant -> 0, InitialState -> Table[1, {i, 1, 20}],
SamplingInterval -> T/32, NumberOfCurves -> 4, ModulatingPulse -> LaurentC}

Modulator[L_][BitSeq_, Opts_] :=
Module[
{x1, x2, x3, x4, x5, x6, state, AccumulatedPhase, seq, AKNState, Curves, Pulse},
x1 = SamplingInterval /. {Opts} /. Options[Modulator];
state = InitialState /. {Opts} /. Options[Modulator];
x3 = StartingQuadrant /. {Opts} /. Options[Modulator];
x4 = SamplingInterval /. {Opts} /. Options[Modulator];
Pulse = ModulatingPulse /. {Opts} /. Options[Modulator];
Curves = (NumberOfCurves /. {Opts} /. Options[Modulator]) - 1;
AccumulatedPhase = x3;
seq = BitSeq;
Table[
AKNState[K] = ANKInitialStateSetup[L][K][state, AccumulatedPhase], {K, 0, Curves}];
x5 := Module[{}, state = Join[{First[seq]}, Drop[state, -1]];
AccumulatedPhase = AccumulatedPhase + First[seq];
seq = Rest[seq];
Table[AKNState[K] = Join[{AKN[L][K][{state, AccumulatedPhase}],
Drop[AKNState[K], -1]}, {K, 0, Curves}];
x6[t_] = Sum[Sum[(J) AKNState[K][[i+1]] Pulse[L][K][t + iT],
{i, 0, LaurentLK[L][K] - 1}], {K, 0, Curves}];
Table[x6[t], {t, 0, T - x4, x4}];
Table[x5, {kk, 1, Length[BitSeq]}] // Flatten]

Options[Receiver] := {StartingQuadrant -> 0,
InitialState -> {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, SamplingInter
ModulatingPulse -> FiltPulse, SyncSample -> 0, NumberOfCurves -> 2};

Receiver[L_][InputSeq_, Opts_] :=
Module[{x1, x2, x3, x4, x5, x6},
x1 = StartingQuadrant /. {Opts} /. Options[Receiver];
x2 = InitialState /. {Opts} /. Options[Receiver];
x3 = SamplingInterval /. {Opts} /. Options[Receiver];
x4 = ModulatingPulse /. {Opts} /. Options[Receiver];
x5 = SyncSample /. {Opts} /. Options[Receiver];
x6 = NumberOfCurves /. {Opts} /. Options[Receiver];
ReceiverProper[L][x1, x2, x3, x4, x5, x6, InputSeq]]

```

```

ReceiverProper[L_][StartingQuadrant_, InitialState_, SamplingInterval_,
ModulatingPulse_, SyncSample_, NumberOfCurves_, InputSeq_] :=
Module[{x1, sgn, ReceivedSeq, ExpectedValue, seq, ReceiveNext, D, J},
x1 = T / SamplingInterval;
ReceivedSeq = Partition[InputSeq, x1] // Transpose // #[[SyncSample + 1]] &;
ExpectedValue =
ModulatingPulse[L][0][(LaurentLK[L][0] / 2) T + SyncSample SamplingInterval];
J = StartingQuadrant;
sgn = 0;
D = {};
seq = ReceivedSeq;
ReceiveNext :=
Module[{x1, x2},
x1 = ((-1)sgn J First[seq]) // Im;
If[Abs[x1 - ExpectedValue] <= Abs[x1 + ExpectedValue],
D = Join[D, {1}]; J = J + 1, D = Join[D, {-1}]; J = J - 1];
seq = Rest[seq]; sgn = Mod[sgn + 1, 2];
Table[ReceiveNext, {i, 1, Length[ReceivedSeq]}];
D]
End[]

EndPackage[]

```

```
Needs["LaurentFunctions`"]
```

```
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
PhaseAngle[L_][t_] -> (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).
```

```
Needs["LaurentNotationTest`"]
```

```
Needs::nocont : Context LaurentNotationTest' was not created when Needs was evaluated.
```

Information on the functions used can be obtained using help.

```
Names["LaurentFunctions`*"]
```

```
{AKN, AlphaKI, ANKInitialStateSetUp, BT, FiltPulse, h, hFiltered, InitialState, J,
LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
NumberOfCurves, PhaseAngle, Receiver, ReceiverProper, S, SamplingInterval,
StartingQuadrant, SyncSample, T, C,  $\Phi$ ,  $\psi$ }
```

```
T :=  $\frac{3}{812500}$ 
BT := 0.3
```

```
ModulationIndex :=  $\frac{1}{2}$ 
```

```
<< ModulatorData.m;
```

```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
-1, 1, -1, -1, 1, -1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, -1,
1, -1, 1, -1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, -1, -1, -1, 1, -1,
1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

We show that it is possible
to build a ~~receiver~~ ^{mobile telephone} to receive ~~the~~
~~pulses~~ that have a signal modulated
using ~~pulse~~ superposition method
of Laurent - but with different pulses
that have other desirable properties
like low bandwidth etc.

$$\frac{3}{812500}$$

$$S_{NT+\Delta T} = \sum_{K=0}^{M-1} \sum_{n'=0}^{L_K-1} J^{A_{KN-n'}} C_{K,n',T+\Delta T}$$

With $L = 8$, and $M = 2$ we can utilise only the main values. It so happens that when $K = 1$, the dominant value occurs at $2.5T$ in the filtered pulse. The three dominant values when $K=0$ occurs at $3.5T$, $4.5T$ and $5.5T$. The value of the pulse at $6.5T$ is smaller than the value of the pulse with $K = 1$ at $2.5T$. Thus the expression becomes

$$S_{NT+\Delta T} = \sum_{K=0}^1 \sum_{n'=3}^5 J^{A_{KN-n'}} C_{K,n',T+\Delta T}$$

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-3}} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{0,N-4}} \text{Pulse}[0][4T + \delta T] + J^{\alpha_{0,N-5}} \text{Pulse}[0][5T + \delta T] + J^{\alpha_{1,N-2}} \text{Pulse}[1][2T + \delta T]$$

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-3}} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{0,N-4}} \text{Pulse}[0][4T + \delta T] + J^{\alpha_{0,N-5}} \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{0,N-2} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T]$$

Given that α_{N-3} has been already decided, we can precalculate the possibilities and store them.

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-5}} (J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T + \delta T] + \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{N-4} + \alpha_{N-3} + \alpha_{N-2} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T])$$

But $J^2 = 1$ and so we get

$$\tilde{S}_{NT+\Delta T} = J^{\alpha_{0,N-5}} (J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[0][3T + \delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T + \delta T] + \text{Pulse}[0][5T + \delta T] + J^{(\alpha_{N-4} + \alpha_{N-3})} \text{Pulse}[1][2T + \delta T])$$

```
LookUpTable[Pulse_, δT_] :=
  Module[{x1, x2, x3, x4},
    x1 = {{-1, -1, -1}, {-1, -1, 1},
      {-1, 1, -1}, {-1, 1, 1}, {1, -1, -1}, {1, -1, 1}, {1, 1, -1}, {1, 1, 1}};
    x2[bitseq_] :=
      (J^bitseq[[3]] + bitseq[[2]]) Pulse[0][3T + δT] + J^bitseq[[3]] Pulse[0][4T + δT] +
      Pulse[0][5T + δT] + J^bitseq[[3]] + bitseq[[1]] Pulse[1][2T + δT];
    {x1, Map[x2[#] &, x1]} // Transpose]
```

```
tab = LookUpTable[FiltPulse[8], 0.5 T]
```

```
{{{-1, -1, -1}, -0.00122183 - 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
  {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, 1, 1}, 0.0869922 + 0.770616 I},
  {{1, -1, -1}, 0.0869922 - 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
  {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, 1, 1}, -0.00122183 + 0.770616 I}}
```

```
Sort[tab, #1[[1, 3]] > #2[[1, 3]] &]
```

```
{{{1, 1, 1}, -0.00122183 + 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
  {{-1, 1, 1}, 0.0869922 + 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
  {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, -1, -1}, 0.0869922 - 0.770616 I},
  {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, -1, -1}, -0.00122183 - 0.770616 I}}
```

```
FiltPulse[8][0][4T + 0.5 T]
```

```
0.770616
```

Now we build a receiver!! First generate the modulated sequence

Options[Modulator]

```
{StartingQuadrant -> 0,
 InitialState -> {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1},
 SamplingInterval ->  $\frac{3}{26000000}$ , NumberOfCurves -> 4, ModulatingPulse -> LaurentC}
```

$$\frac{3}{26000000} / T$$

$$\frac{1}{32}$$

```
tom2 = Modulator[L][RandomBitSeq, NumberOfCurves -> 2, ModulatingPulse -> FiltPulse];
```

```
ListPlot[{Re[tom2], Im[tom2]} // Transpose, PlotJoined -> True]
```

- Graphics -

RandomBitSeq

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

```
Receiver[L][tom2, StartingQuadrant -> 4]
```

```
{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
 -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1,
 1, 1, 1, -1, 1, -1, 1, -1, 1, 1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1,
 -1, -1, 1, -1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1}
```

We now calculate the BER given the $\frac{E_b}{N_0}$ and $L = 8$. We write all the possible terms

$$\begin{aligned} J^{a_0, N-5} & (J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N)} \text{Pulse}[0][\delta T] + \\ & J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1})} \text{Pulse}[0][T+\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2})} \text{Pulse}[0][2T+\delta T] + \\ & J^{(a_{N-4}+a_{N-3})} \text{Pulse}[0][3T+\delta T] + J^{a_{N-4}} \text{Pulse}[0][4T+\delta T] + \text{Pulse}[0][5T+\delta T] + \\ & J^{(-a_{N-5})} \text{Pulse}[0][6T+\delta T] + J^{(-a_{N-5}-a_{N-6})} \text{Pulse}[0][7T+\delta T] + \\ & J^{(-a_{N-5}-a_{N-6}-a_{N-7})} \text{Pulse}[0][8T+\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N-a_{N-1})} \text{Pulse}[1][\delta T] + \\ & J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}-a_{N-2})} \text{Pulse}[1][T+\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1})} \text{Pulse}[1][2T+\delta T] + \\ & J^{(a_{N-4}+a_{N-3}-a_{N-4})} \text{Pulse}[1][3T+\delta T] + J^{(a_{N-4}-a_{N-5})} \text{Pulse}[1][4T+\delta T] + \\ & J^{(-a_{N-6})} \text{Pulse}[1][5T+\delta T] + J^{(-a_{N-5}+a_{N-7})} \text{Pulse}[1][6T+\delta T]) \end{aligned}$$

We select the imaginary

$$\begin{aligned} J^{a_0, N-5} & (J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N)} \text{Pulse}[0][\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2})} \text{Pulse}[0][2T+\delta T] + \\ & J^{a_{N-4}} \text{Pulse}[0][4T+\delta T] + J^{(-a_{N-5})} \text{Pulse}[0][6T+\delta T] + \\ & J^{(-a_{N-5}-a_{N-6}-a_{N-7})} \text{Pulse}[0][8T+\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}-a_{N-2})} \text{Pulse}[1][T+\delta T] + \\ & J^{(a_{N-4}+a_{N-3}-a_{N-4})} \text{Pulse}[1][3T+\delta T] + J^{(-a_{N-6})} \text{Pulse}[1][5T+\delta T]) \end{aligned}$$

```
ModulationValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][ $\delta T$ ] :=
(1. I) x0 + x1 + x2 + x3 + x4 Pulse[0][ $\delta T$ ] + (1. I) x2 + x3 + x4 Pulse[0][2T +  $\delta T$ ] +
(1. I) x4 Pulse[0][4T +  $\delta T$ ] + (1. I) x5 Pulse[0][6T +  $\delta T$ ] +
(1. I) x5 + x6 + x7 Pulse[0][8T +  $\delta T$ ] + (1. I) x1 + x3 + x4 Pulse[1][T +  $\delta T$ ] +
(1. I) x3 Pulse[1][3T +  $\delta T$ ] + (1. I) x6 Pulse[1][5T +  $\delta T$ ]
```

-0.754091 I

Module[{bit, n},

```
ConvertNext := (bit = Mod[n, 2]; n = Floor[n/2]; bit);
```

```
ConvertToBitSeq[8][2]
```

```
Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&]
```

```
{-1, -1, -1, -1, 1, -1, -1, -1},
{-1, -1, -1, -1, 1, -1, -1, 1}, {-1, -1, -1, -1, 1, -1, -1, -1},
{-1, -1, -1, -1, 1, -1, 1, 1}, {-1, -1, -1, -1, 1, 1, -1, -1},
{-1, -1, -1, -1, 1, 1, -1, 1}, {-1, -1, -1, -1, 1, 1, 1, -1},
{-1, -1, -1, -1, 1, 1, 1, 1}, {-1, -1, -1, 1, 1, -1, -1, -1},
{-1, -1, -1, 1, 1, -1, -1, 1}, {-1, -1, -1, 1, 1, -1, 1, -1},
{-1, -1, -1, 1, 1, -1, 1, 1}, {-1, -1, -1, 1, 1, 1, -1, -1}, {-1, -1, -1, 1, 1, 1, 1, -1},
{-1, -1, -1, 1, 1, 1, 1, 1}, {-1, -1, -1, 1, -1, 1, -1, -1},
{-1, -1, 1, -1, 1, -1, 1, 1}, {-1, -1, 1, -1, 1, 1, 1, -1}, {-1, -1, -1, 1, 1, 1, 1, -1},
{-1, -1, 1, -1, 1, 1, 1, -1}, {-1, -1, 1, 1, -1, 1, 1, 1}, {-1, -1, 1, 1, 1, -1, -1, -1},
{-1, -1, 1, 1, 1, -1, -1, 1}, {-1, -1, 1, 1, 1, -1, 1, -1}, {-1, -1, 1, 1, 1, -1, 1, 1},
{-1, -1, 1, 1, 1, 1, -1, -1}, {-1, -1, 1, 1, 1, 1, -1, 1}, {-1, -1, 1, 1, 1, 1, 1, -1},
{-1, -1, 1, 1, 1, 1, 1, 1}, {-1, 1, -1, -1, 1, -1, -1, -1}, {-1, 1, -1, -1, 1, -1, -1, 1},
{-1, 1, -1, -1, 1, 1, -1, -1}, {-1, 1, -1, -1, 1, 1, -1, 1}, {-1, 1, -1, -1, 1, 1, 1, -1},
{-1, 1, -1, -1, 1, 1, 1, -1}, {-1, 1, -1, 1, 1, 1, -1, -1}, {-1, 1, -1, 1, 1, 1, 1, -1},
{-1, 1, -1, 1, 1, 1, 1, 1}, {-1, 1, -1, 1, 1, 1, 1, 1}, {-1, 1, -1, 1, 1, 1, 1, 1},
{-1, 1, 1, -1, 1, 1, 1, -1}, {-1, 1, 1, -1, 1, 1, 1, 1}, {-1, 1, 1, 1, -1, 1, 1, -1},
{-1, 1, 1, 1, -1, 1, 1, 1}, {-1, 1, 1, 1, -1, 1, 1, 1}, {-1, 1, 1, 1, 1, -1, 1, 1},
{-1, 1, 1, 1, 1, 1, -1, -1}, {-1, 1, 1, 1, 1, 1, -1, 1}, {-1, 1, 1, 1, 1, 1, 1, -1},
{-1, 1, 1, 1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, -1, -1, -1}, {1, -1, -1, -1, 1, -1, -1, 1},
{1, -1, -1, -1, 1, -1, 1, -1}, {1, -1, -1, -1, 1, 1, -1, 1}, {1, -1, -1, -1, 1, 1, 1, -1},
{1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, 1, 1, 1, 1, -1}, {1, -1, -1, 1, 1, 1, 1, 1},
{1, -1, 1, -1, 1, -1, 1, -1}, {1, -1, 1, -1, 1, 1, -1, -1}, {1, -1, 1, -1, 1, 1, -1, 1},
{1, -1, 1, -1, 1, 1, 1, -1}, {1, -1, 1, -1, 1, 1, 1, 1}, {1, -1, 1, 1, -1, 1, 1, -1},
{1, -1, 1, 1, -1, 1, 1, 1}, {1, -1, 1, 1, 1, -1, 1, -1}, {1, -1, 1, 1, 1, -1, 1, 1},
{1, -1, 1, 1, 1, 1, -1, -1}, {1, -1, 1, 1, 1, 1, -1, 1}, {1, -1, 1, 1, 1, 1, 1, -1},
{1, -1, 1, 1, 1, 1, 1, 1}, {1, 1, -1, -1, 1, -1, -1, -1}, {1, 1, -1, -1, 1, -1, -1, 1},
{1, 1, -1, -1, 1, 1, -1, -1}, {1, 1, -1, -1, 1, 1, 1, -1}, {1, 1, -1, -1, 1, 1, 1, 1},
{1, 1, -1, 1, 1, -1, -1, -1}, {1, 1, -1, 1, 1, -1, -1, 1}, {1, 1, -1, 1, 1, -1, 1, 1},
{1, 1, -1, 1, 1, 1, -1, -1}, {1, 1, -1, 1, 1, 1, -1, 1}, {1, 1, -1, 1, 1, 1, 1, -1},
{1, 1, 1, -1, 1, 1, 1, -1}, {1, 1, 1, 1, -1, 1, -1, -1}, {1, 1, 1, 1, -1, 1, -1, 1},
{1, 1, 1, 1, 1, -1, 1, -1}, {1, 1, 1, 1, 1, -1, 1, 1}, {1, 1, 1, 1, 1, 1, -1, -1},
{1, 1, 1, 1, 1, 1, -1, 1}, {1, 1, 1, 1, 1, 1, 1, -1}, {1, 1, 1, 1, 1, 1, 1, 1}
```



```

x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
  Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
  (-I)

{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}

```

```

Map[(1/2 - σ/2 Erf[ $\frac{\#}{\sigma}$ ])&, x1] // Apply[Plus, #]& // #/128 &;

```

```

Ber[σ_] := Evaluate[%48]

```

```

Ber[0.01]

```

```

0.495

```

```

Erf[Infinity]

```

```

1

```

```

D[Erf[x], x]

```

$$\frac{2E^{-x^2}}{\sqrt{\pi}}$$

```

?Erf

```

```

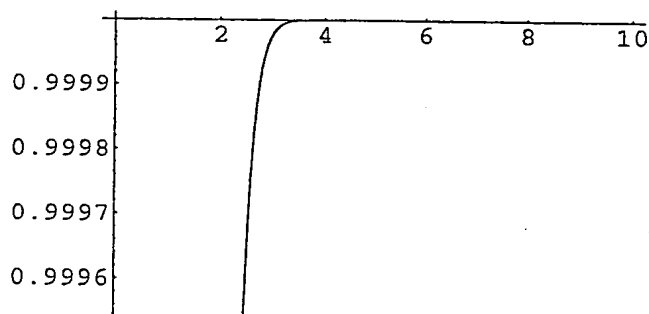
Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error
function erf(z1) - erf(z0).

```

```

Plot[Erf[x], {x, 0, 10}]

```



```

- Graphics -

```

```

?FiltPulse

```

```

Global FiltPulse

```

T := 3 / 812500

Null

?? T

This is the symbol period

?? T

This is the symbol period

T := 3 / 812500

Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]

$$J^{(a_{N-4}+a_{N-3}+a_{N-2}+a_{N-1}+a_N)} \text{Pulse}[0][\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-2})} \text{Pulse}[0][2T+\delta T] +$$

$$J^{a_{N-4}} \text{Pulse}[0][4T+\delta T] + J^{(-a_{N-5})} \text{Pulse}[0][6T+\delta T] +$$

$$J^{(-a_{N-5}-a_{N-6}-a_{N-7})} \text{Pulse}[0][8T+\delta T] + J^{(a_{N-4}+a_{N-3}+a_{N-1})} \text{Pulse}[1][T+\delta T] +$$

$$J^{(a_{N-3})} \text{Pulse}[1][3T+\delta T] + J^{(-a_{N-6})} \text{Pulse}[1][5T+\delta T] /. \{a_N \rightarrow x0, a_{N-1} \rightarrow x1,$$

$$a_{N-2} \rightarrow x2, a_{N-3} \rightarrow x3, a_{N-4} \rightarrow x4, a_{N-5} \rightarrow x5, a_{N-6} \rightarrow x6, a_{N-7} \rightarrow x7\}$$

$$(1. I)^{x0+x1+x2+x3+x4} \text{Pulse}[0][\delta T] + (1. I)^{x2+x3+x4} \text{Pulse}[0][2T+\delta T] +$$

$$(1. I)^{x4} \text{Pulse}[0][4T+\delta T] + (1. I)^{-x5} \text{Pulse}[0][6T+\delta T] + (1. I)^{-x5-x6-x7} \text{Pulse}[0][8T+\delta T] +$$

$$(1. I)^{x1+x3+x4} \text{Pulse}[1][T+\delta T] + (1. I)^{x3} \text{Pulse}[1][3T+\delta T] + (1. I)^{-x6} \text{Pulse}[1][5T+\delta T]$$

$$J^{(a_{N-3})} \text{Pulse}[1][3T+\delta T] /. \{a_N \rightarrow x0, a_{N-1} \rightarrow x1, a_{N-2} \rightarrow x2, a_{N-3} \rightarrow x3,$$

$$a_{N-4} \rightarrow x4, a_{N-5} \rightarrow x5, a_{N-6} \rightarrow x6, a_{N-7} \rightarrow x7\}$$

$$(1. I)^{x3} \text{Pulse}[1][3T+\delta T]$$

$$(1. I)^{x3} \text{Pulse}[1][3T+\delta T]$$

$$(1. I)^{x3} \text{Pulse}[1][3T+\delta T]$$

? J

J = e^{iδ}